Temecula Valley Math Competition

Multiple Choice (60 minutes)

April 13, 2024

INSTRUCTIONS

1. DO NOT OPEN TEST BOOKLET UNTIL INSTRUCTED TO DO SO.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the scantron with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive $+4$ points for each correct answer, -1 point for each problem answered incorrectly, and $+0$ points for each answer left blank.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. When your proctor gives the signal, begin working on the problems. You will have 60 minutes to complete the test.
7. HAVE FUN!

1. What is $\sqrt{1111111111 - 22222}$?

   (A) 33333 (B) 3333 (C) 11211 (D) 1111 (E) 11111

2. Which of the following is the largest number?

   (A) $\frac{4132024}{38}$ (B) 5^{10} (C) 10^5 (D) 3.7×10^5 (E) 3^{3^3}

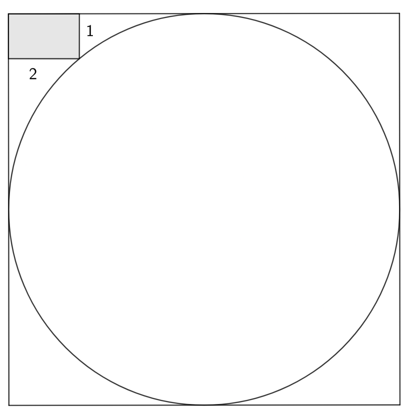
3. How many 4 digit positive integers are divisible by both 5 and 6?

   (A) 300 (B) 299 (C) 160 (D) 159 (E) 150

4. Emmy drove from Temecula to Los Angeles going 70 miles/hour, unless she was stuck in traffic, in which case she went 5 miles/hour. Given that Emmy's average speed was 35 miles/hour, what fraction of the time was she stuck in traffic?

   (A) $\frac{7}{13}$ (B) $\frac{7}{12}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{1}{3}$

5. A circle is inscribed in a square such that a 2×1 rectangle fits in the corner, as shown in the diagram below. What is the side length of the square?



   (A) $5\sqrt{3}$ (B) 5 (C) 8 (D) 10 (E) 20

6. Pierre tosses 3 fair coins and René tosses 4 fair coins. What is the probability that René gets strictly more heads than Pierre?

   (A) $\frac{5}{8}$ (B) $\frac{7}{16}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$ (E) $\frac{1}{4}$

7. A triangle in the xy-plane has two vertices, at $(7, 10)$, and $(-2, 5)$. If its centroid is located at $(4, 2)$, then what are the coordinates of the third vertex?

   (A) $(3, -1)$ (B) $(3, 1)$ (C) $(7, -9)$ (D) $(9, -7)$ (E) $\left(8, -\frac{10}{3}\right)$

8. Suppose T, V, M, C are numbers satisfying the equations

$$\begin{aligned} T + V + M + C &= 2 \\ T - V + M - C &= 0 \\ T - V - M - C &= 2 \\ T + V + M - C &= 4. \end{aligned}$$

   What is the value of the product $T \cdot V \cdot M \cdot C$?

   (A) 0 (B) -2 (C) 2 (D) -4 (E) 4

9. The sum of the lengths of all the edges of a rectangular prism equals 156. What is the maximum possible surface area of the rectangular prism?

   (A) 1014 (B) 864 (C) 800 (D) 464 (E) 442

10. $\sqrt{24 + \sqrt{176}}$ can be written as $\sqrt{n} + \sqrt{m}$ for some integers $n, m > 0$. What is $n + m$?

    (A) 16 (B) 20 (C) 24 (D) 30 (E) 32

11. An n-bit string is a string of all zeros and ones with length n. How many 10-bit strings contain the substring "0000" exactly once?

    (A) 128 (B) 136 (C) 138 (D) 151 (E) 152

12. Ada sends Alan a message containing the name of the person she wants to marry. To keep it secret, she cyclically shifts each letter by a predetermined number $0 < k < 26$. For example, if $k = 5$ and the name is `XAVIER`, she would send `CFANJW`.
    Alan receives the message `JVYYVNZ`. Who does Ada want to marry?

    (A) `MATTHEW` (B) `PHILLIP` (C) `ABRAHAM` (D) `WILLIAM` (E) `JEFFERY`

13. Evaluate $\sin\left(\sin^{-1}\frac{3}{5} + \tan^{-1} 2\right)$

    (A) $\frac{11\sqrt{5}}{25}$ (B) $\frac{-5\sqrt{5}}{25}$ (C) $\frac{17}{20}$ (D) $\frac{22}{25}$ (E) $\frac{6\sqrt{5}}{25}$

14. Let $a\Theta b = \left(\frac{1}{a}+a\right)^b$. Find the value of the coefficient of $c^2\Theta 1$ in the expansion of $c\Theta 6$.

    (A) 1 (B) 6 (C) 15 (D) 20 (E) 35

15. What is the area of the circle inscribed in a triangle with side lengths 5 ,12, and 13?

    (A) $\frac{13\pi}{12}$ (B) 9π (C) $\frac{13\pi}{5}$ (D) $\frac{12\pi}{5}$ (E) 4π

16. In $\triangle ABC$, D is the midpoint of BC and E is the trisection point of AC nearer to A (i.e., $AE : EC = 1 : 2$). Let $G = BE \cap AD$. Find $AG : GD$.

    (A) $2 : 3$ (B) $1 : 2$ (C) $2 : 1$ (D) $1 : 1$ (E) $3 : 1$

17. What is the maximum number of $1 \times 1 \times 4$ blocks that can fit inside a $6 \times 6 \times 6$ cube?

    (A) 50 (B) 51 (C) 52 (D) 53 (E) 54

18. What is the number of integers $1 \leq n \leq 2024$ where the last two digits of n^3 are 11?

    (A) 30 (B) 20 (C) 21 (D) 29 (E) 40

19. How many positive integer triples (a, b, c) are there such that $a+b+c = 42$, and a, b, c form the side lengths of a (possibly degenerate) triangle?
    That is, $a + b \geq c$, $b + c \geq a$, and $c + a \geq b$.

    (A) 191 (B) 200 (C) 210 (D) 231 (E) 250

20. Find the sum of all $n > 0$ such that $1/n$ can be written as $0.\overline{abc} = 0.abcabcabc\ldots$

    (A) 510 (B) 511 (C) 1507 (D) 1510 (E) 1520

21. Mr. Pascal gambles \$300 on a game that costs \$100 to play. Each round he doubles his \$100 with probability 1/3, and loses \$100 with probability 2/3. He plays the game repeatedly until he either has \$600 total, or he loses all his money.
    What is the probability that he loses all his money?

    (A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{8}{9}$ (D) $\frac{7}{9}$ (E) $\frac{6}{7}$

22. Suppose $f : (1, \infty) \to \mathbb{R}$ is a continuous function that satisfies

$$f\left(\frac{x}{x-1}\right) = \frac{f(x)}{2} + 4x.$$

Find the sum of all $x > 1$ such that $f(x) = 24$.

(A) 3 (B) $\frac{19}{3}$ (C) 4 (D) 8 (E) $2\sqrt{7}$

23. Find the value of the sum

$$\sum_{k=0}^{253} \left\lfloor \frac{17k+1}{254} \right\rfloor .$$

(A) 2044 (B) 2025 (C) 2024 (D) 577 (E) 578

24. Let P be a degree n polynomial such that $P(k) = 2^k$ for all integers $0 \le k \le n$. What is $P(n+1)$?

(A) 2^{n+1} (B) $2^{n+1} - 1$ (C) $(-2)^{n+1}$ (D) $(-1)^n(2^{n+1} - 1)$ (E) 0

25. Define an integer sequence by $f(0) = 0$ and $f(n) = n - f(f(n-1))$ for $n > 0$. What is $f(1000)$?

(A) 501 (B) 512 (C) 513 (D) 617 (E) 618