Temecula Valley Math Competition

Team Round (60 minutes)

April 13, 2024

Names: ____

School: ____

INSTRUCTIONS

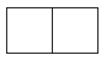
1. DO NOT OPEN TEST BOOKLET UNTIL INSTRUCTED TO DO SO.
2. Print your team members' names and school in the spaces provided above.
3. Ask a test proctor if you need any clarifications on a problem.
4. While you can (and should) work with your teammates on the problems, you will submit only one answer sheet for your whole team. DO NOT submit multiple attempts for the same problem.
5. This section consists of 4 problems, each worth 10 points. Put all work towards a solution in the space following the problem statement and the following blank page. If you use extra sheets of paper, write your names and the problem number at the top and attach them to this packet. Draw $\boxed{\textit{boxes}}$ around your final answers.
6. SCORING: You will receive +10 points for a correct answer OR a maximum of +2 points for work towards an answer. You DO NOT have to write rigorous proofs.
7. HAVE FUN

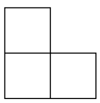
1. Fill in the empty cells of the 5×5 grid with the numbers $1, 2, 3, 4, 5$ such that each row and column contains each number exactly once. Additionally, the numbers must satisfy the greater-than relations indicated by $<$ signs between cells.

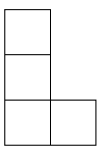
3							$<$	
		1						
$\vee$								
				4				
						1		
			$<$					5

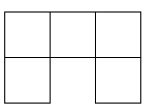
2. Alice and Bob play a game on an infinite square grid given a polyomino **P** (a *polyomino* is a connected subset of squares in the grid).
   On each turn, a player claims an empty square and writes their initial in it. Alice wins if she can make a congruent copy of **P** with her squares (i.e. possibly reflected or rotated), while Bob tries to prevent this. Alice moves first.

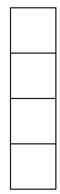
   For each **P** below, write *Alice* if Alice can win the game, or *Bob* if Bob can prevent Alice from winning.

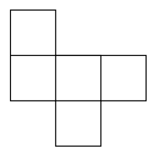
   Example:  $\boxed{\textit{Alice}}$

   (a) 

   (b) 

   (c) 

   (d) 

   (e) 

3. Let $\triangle ABC$ be a triangle with $\angle A = 60°$, $\angle B = 45°$, $\overline{AC} = 2$, and $\overline{BC} = \sqrt{6}$.
A ball is launched from point D on side AB, bounces off point E on BC, then bounces off point F on AC, and finally returns to D. In other words, the trajectory of the ball forms an inscribed triangle $\triangle DEF$.
The ball bounces such that $\angle ADF = \angle BDE$, $\angle CEF = \angle BED$, $\angle AFD = \angle CFE$.

   Determine the values of the following:

   (a) $\angle DEF$

   (b) $\angle EFD$

   (c) $\angle FDE$

   (d) the perimeter of $\triangle DEF$

4. Find an $x > 0$ such that $x^2 + 5$, x^2, and $x^2 - 5$ are squares of rational numbers.