Temecula Valley Math Competition

Team Round (60 minutes)

March 18, 2023

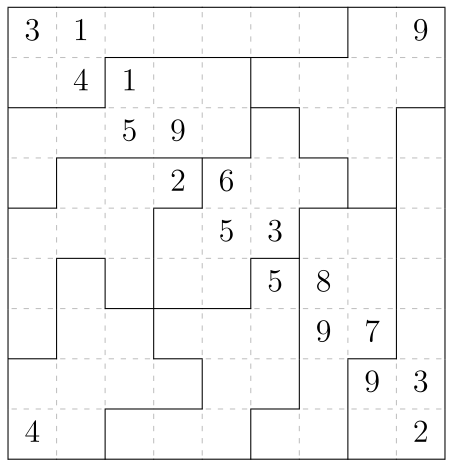
Names: ____

School: ____

INSTRUCTIONS

1. DO NOT OPEN TEST BOOKLET UNTIL INSTRUCTED TO DO SO.
2. Print your team members' names and school in the spaces provided above.
3. While you can (and should) work with your teammates on the problems, you will submit only one answer sheet for your whole team. DO NOT submit multiple attempts for the same problem.
4. This section consists of 4 problems, each worth 10 points. Put all work towards a solution in the space following the problem statement and the following blank page. If you use extra sheets of paper, write your names and the problem number at the top and attach them to this packet.
5. SCORING: You are graded based on the correctness, completeness, and clarity of your solutions. Excepting numeric answers, all answers must be rigorously justified. Clearly state any theorems that you use.

1. In jigsaw sudoku, the goal is to place the digits 1 to 9 in a 9×9 board so that each digit appears *exactly once* in each row, column, and jigsaw piece (jigsaw pieces are the shapes outlined in black). Solve the board below by filling in the blank squares.



2. Suppose that a triangle has side lengths a, b, c and area A. Prove the following inequality, and find the conditions for which equality is achieved.

$$a^2 + b^2 + c^2 \geq 4\sqrt{3}A.$$

3. We say an integer has a *hexary* expansion if it can be represented as a sum of terms of the form $2^n \cdot 3^m$, $n, m \geq 0$ such that for every pair of terms, one term is not a divisor of the other. For example, $19 = 2^2 + 2^1 3^1 + 3^2$ is a valid hexary expansion.

   Prove or disprove: Every positive integer has a hexary expansion.

4. (a) n^2 cells are arranged in a $n \times n$ square grid. Initially, some cells are infected and the rest are healthy. After one minute, every healthy cell with at least two infected neighbors (horizontally or vertically) becomes infected. Infected cells stay infected. This process continues until the infection cannot reach more cells. Find, with proof, the minimum number of initially infected cells that guarantees all cells will eventually become infected.

   (b) Now consider n^3 cells in a $n \times n \times n$ cube. The infection spreads the same way, but now cells can have neighbors in six directions. Find the minimum number of initial infected cells that guarantees all cells will eventually become infected.