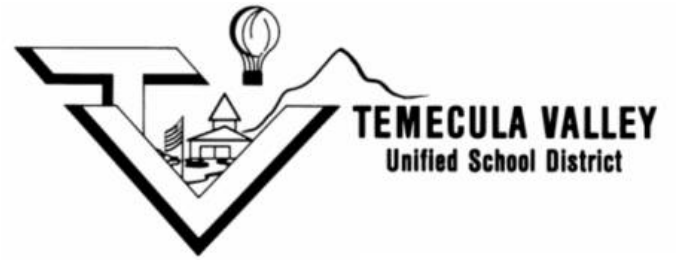


2018 Temecula Valley
High School Math Competition
Multiple Choice Test (60 mins)
February 10th, 2018

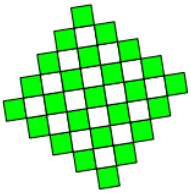
Instructions:

1. DO NOT OPEN TEST BOOKLET UNTIL INSTRUCTED TO DO SO.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the scantron provided with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive +4 points for each correct answer, -1 point for each problem answered incorrectly, and +0 points for each answer left blank.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 60 minutes to complete the test.

1. What is the sum of $1 + 2 + 3 + 4 + ... + 9996 + 9997 + 9998 + 9999 + 10000$?

A.) $500,500$ **B.)** $50,005,000$ **C.)** $500,000$ **D.)** $5,000,000$ **E.)** $5,000,050,000$

2. How many more colored unit squares are there than white unit squares in the diagram?



A.) 8 **B.)** 9 **C.)** 25 **D.)** 16 **E.)** 10

3. Three points lie in a plane, not all on the same line. How many lines can be drawn in that same plane, which are equidistant from these points? (The distance from a line to a point P is the shortest distance from P to any point on the line).

A.) 3 **B.)** 2 **C.)** 1 **D.)** 0 **E.)** 42

4. Calculate

$$\sqrt{\binom{3}{2} + \binom{4}{2} + \binom{4}{2} + \binom{5}{2}}$$

A.) 25 **B.)** $\sqrt{24}$ **C.)** 4 **D.)** 5 **E.)** 42

5. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ be arithmetic sequences such that $a_1 + b_1 + c_1 = 1$ and $a_0 + b_0 + c_0 = 0$. What is $a_{2018} + b_{2018} + c_{2018}$?

A.) 2015 **B.)** 2018 **C.)** 2017 **D.)** 1 **E.)** 0

6. Let $F_1 = F_2 = 1$, $F_k = F_{k-1} + F_{k-2}$ for $k \geq 3$. Find the value of the sum

$$F_1 + F_2 + ... + F_{10}$$

A.) 90 **B.)** 42 **C.)** 89 **D.)** 144 **E.)** 143

7. There are 4 black cats and 2 orange cats in bag A. There are 3 black and 5 orange cats in bag B. A bag is randomly selected and a cat chosen from the bag. The cat is black. What is the probability that it came from bag A?

A.) $\frac{1}{2}$ **B.)** $\frac{16}{25}$ **C.)** $\frac{1}{3}$ **D.)** $\frac{25}{16}$ **E.)** $\frac{1}{4}$

8. An integer n such that $10 \leq n \leq 99$ is chosen at random. What is the probability that at least one digit of n is 7?

A.) $\frac{1}{7}$ **B.)** $\frac{1}{5}$ **C.)** $\frac{1}{9}$ **D.)** $\frac{2}{5}$ **E.)** $\frac{1}{6}$

9. In a math competition, $\frac{1}{2}$ of the participants like math, $\frac{1}{3}$ of the participants don't like math, and $\frac{1}{6}$ love math. Given that there were 40 more people who either like math or don't like math than those who love math, how many participants were in the competition?

A.) 42 **B.)** 30 **C.)** 60 **D.)** 45 **E.)** 50

10. Find the square of the difference between the first 100 positive even integers and the first 100 positive odd integers?

A.) 100 **B.)** 0 **C.)** $2,500$ **D.)** $10,000$ **E.)** 1

11. The polynomial $P(x) = x^4 - 14x^3 + 99x^2 + ax + b$ has two double roots. What is the value of $a + b$?

A.) 400 **B.)** -800 **C.)** 800 **D.)** 450 **E.)** -450

12. Find the number of integers n with $n \geq 2$ such that the remainder when 2019 is divided by n is equal to the remainder when n is divided by 3.

A.) 4 **B.)** 0 **C.)** 1 **D.)** 2 **E.)** 3

13. In triangle ABC, $AB = AC$ and $m\angle A = 80°$ If points D, E and F lie on sides BC, AC and AB, respectively, and $CE = CD$ and $BF = BD$, then $m\angle EDF$ equals

A.) $30°$ **B.)** $40°$ **C.)** $50°$ **D.)** $65°$ **E.)**none of these

14. Define the sequence $\{x_n\}$ as follows: $x_0 = 0$, $x_1 = 1$, $x_k = 2x_{k-1} - x_{k-2} + 2$ for $k \geq 2$. What is the value of $x_{1010} - x_{1009}$?

A.) 2018^2 **B.)** 2019^2 **C.)** 2017 **D.)** 2018 **E.)** 2019

15. How many distinct nonempty subsets can be made from the following set $\{x_1, x_2, x_3, ..., x_{10}\}$?

A.) 1023 **B.)** 1024 **C.)** 511 **D.)** 512 **E.)** 10

16. What is the units digit of $3^{2018} - 4^{2018}$?

A.) 7 **B.)** 1024 **C.)** 2 **D.)** 3 **E.)** 5

17. The quadrilateral $ABCD$ has consecutive sidelengths 8, 15, and 12 and is inscribed in a circle with circumference 17π. Given that AC is the diameter of the circle, what is the length of BD?

A.) $\frac{96 + 15\sqrt{145}}{17}$ **B.)** $\sqrt{145}$ **C.)** $\frac{96}{17}$ **D.)** 42 **E.)** $\frac{15 + 96\sqrt{145}}{17}$

18. How many ways can 492 identical cats be partitioned into piles of either three or four cats so that every cat is in some pile?

A.) 123 **B.)** 41 **C.)** 42 **D.)** 43 **E.)** 122

19. A point P is selected at random on line segment AB, which has a midpoint M. What is the probability that three line segments with lengths AP, PB, and AM can form a triangle?

A.) $\frac{1}{4}$ **B.)** $\frac{1}{3}$ **C.)** $\frac{2}{3}$ **D.)** $\frac{1}{2}$ **E.)** 0

20. What is the sum of all real roots of $f(x) = x^5 + 4x^4 + x^3 - x^2 - 4x - 1$?

A.) 10 **B.)** -6 **C.)** 6 **D.)** 3 **E.)** -3

21. The triangular pyramid $ABCD$ has an equilateral base ABC with $AB = BC = CA = 1$. If $AD = BD = DC$ and $m\angle ADB = m\angle BDC = m\angle ADC = 90°$, what is the volume of $ABCD$?

A.) $\frac{\sqrt{3}}{2}$ **B.)** $24\sqrt{2}$ **C.)** $\frac{2\sqrt{3}}{4}$ **D.)** $\frac{\sqrt{2}}{24}$ **E.)** 42

22. Find the value of x if

$$\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3$$

A.) 7 **B.)** 80 **C.)** $\sqrt{80}$ **D.)** 8 **E.)** $\sqrt{27}$

23. Given that the expression on the left exists, find the real positive solution to the equation

$$\sqrt{x + 2\sqrt{x + 2\sqrt{x + \cdots + 2\sqrt{x + 2\sqrt{2x + x}}}}} = x$$

A.) 3 **B.)** 2 **C.)** $\sqrt{2}$ **D.)** $\sqrt{3}$ **E.)** $\sqrt{2} - 1$

24. What is the coefficient of x^5 in $(1 + x^2 + x^3)^7$?

A.) 1 **B.)** 42 **C.)** 7 **D.)** 21 **E.)** 168

25. Find the value of the sum

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{8 \cdot 4^n - 6 \cdot 2^n + 1}$$

A.) $\frac{1}{3}$ **B.)** $\frac{1}{2}$ **C.)** $\frac{1}{6}$ **D.)** $\frac{2}{3}$ **E.)** The sum diverges