



**2018 Temecula Valley  
High School Math Competition**

Free Response Questions (60 mins)

February 10th, 2018

Please print your Name, School, and ID:

**Name:** \_\_\_\_\_

**School:** \_\_\_\_\_ **ID:** \_\_\_\_\_

**Instructions:**

1. DO NOT OPEN TEST BOOKLET UNTIL INSTRUCTED TO DO SO.
2. This section consists of 4 problems, each worth 25 points. These problems are “essay” style questions. You should put all work towards a solution in the space following the problem statement. You should show all work and justify your responses as best as you can
3. Scoring is based on the progressed you have made in understanding and solving the problem. The clarity and elegance of your response is an important part of the scoring. You may use the back side of each sheet to continue your solution, but be sure to notify the reader.
4. Give this entire exam to a proctor when you have completed the test to your satisfaction.

1. Given 50 distinct positive integers less than 100, prove that the sum of two of those integers is exactly 99.

2. What is the number of positive integers  $n$  such that a convex regular polygon with  $n$  sides has interior angles with integer measures in degrees? (For example, a regular 6-gon has interior angles with measure  $120^\circ$ )

3. Prove that  $\sqrt{2} + \sqrt{3} + \sqrt{5}$  is an irrational number.

4. Determine all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  satisfying

$$xf(y) + yf(x) = (x + y)f(x^2 + y^2)$$

for all  $x, y \in \mathbb{N}$